

# Limiting the State Space Explosion as Taking Dynamic Issues into Account in Network Modelling and Analysis

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## Extended Abstract

### 1 Background

Dependability [1] is a very important aspect of a computing or communication system. Modelling and analysis is a prominent means for system dependability evaluation [2]. The existing dependability modelling and analysis work for large systems like communication networks focus on the static issues (such as connectivity), mainly due to the increasing complexity and thus the limitations of insufficient computation capability. However, actually the communication networks are dynamically changing their characteristics, such as topology, routes, traffic loads, bandwidth, etc. Therefore, there are a lot of dynamic issues related to dependability at various levels (component, system, service, user, etc.) inside the networks, to name a few, changing network characteristics, differentiation of dependability (e.g. end user or service requirements, actor viewpoints), and dynamic multiparty interactions (e.g. interrelationship of multiple parties, dynamic interaction). In order to reflect better the truth and get closer to the dynamic and complex reality with respect to dependability in the networking context, those dynamic issues should be definitely taken into account in network modelling and analysis. This is our objective to extend the existing dependability modelling methodologies so as to deal with the new challenges in the arising dynamic environment.

### 2 Modelling Approaches

In general, network models can be classified as either structural or behavioral [3]. Structural models focus on the organization of the network and its components, while behavioral models focus on the dynamics of the network and the traffic workload. Most of existing dependability models (e.g. [4], [5], [6], [7]) fall into the category of structural models with focus on the connectivity measures. With these models, the system or service is thought to be dependable as long as it is connected. This consideration does not include the forementioned dynamics in the network, for example, the time

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delay from a failure occurred in the current route until a switchover of the traffic to the later found new route, and the traffic handling issues because of the link capacity constraints. To overcome the limitations of the structural models, behavioral models are needed to incorporate the behavioral properties of the system. With behavioral models, the system or service is considered as dependable when not only the connection exists but also some of its behavior aspects are reflected, for instance, a certain degree of requirements from the end users are met.

### 3 State Space Explosion Problem

As a result to incorporate the dynamic issues into the modelling, there will be some new challenges to the modelling process. In order to deal with the dynamic issues in the networks, the first thing is to clearly specify them in the model. Hence, a set of supplementary information is needed to accomplish this, for example the network topology, component information, route information, link state database, etc. These supplementary information, if are specified improperly, could also cause the explosion of the final state space and become a big problem to the model solution. Furthermore, as traffic flows throughout the network, a lot of decisions (e.g. routing, traffic mapping) should be made in the dynamic environment and time. In the decision-making process, many judgments of conditions based on the currently available supplementary information are needed to be made as a reflection in the dependability model. By allowing more supplementary information and condition judgments in the dependability model, the entire state space will definitely be enlarged, and it will also add more complexity to the modelling process. In the modelling with existing dependability modelling methodologies that deal with static issues traditionally, the state space explosion problem arises typically when the number of components in the system is relatively large. For instance, the number of states in a Markov model usually grows exponentially with the number of components in the system. However, by incorporating the more dynamic issues into the network model, the number of states would also explode very largely with the higher level of incorporated dynamics. Therefore, the state space explosion problem becomes even worse as taking dynamic issues into account in network modelling and analysis. In this case, even for a moderate size system, the number of states could also possibly grow to be prohibitively large as the relatively more dynamics of the system is included in the modelling. This problem causing by state space explosion becomes a very big challenge with respect to not only generate the entire state space but also to obtain an (reasonable) accurate solution result. The storage space will limit the generation of the entire state space, meanwhile the computation capability will limit the obtainment of an exact result.

### 4 Mathematical Techniques

Several techniques, if applicable, could be applied to overcome or reduce the difficulties, for example, state lumping [8], approximation methods by state truncation and decomposition/aggregation [9], or bounding methods [10]. However, most of the work are done at the state-level and not directly applicable and appropriate to deal with the dynamic issues that require higher-level model representations. Some existing work at the model-level (e.g. model-level or compositional lumping, decomposition/aggregation) try to explore certain properties (e.g. equivalence, structural symmetry, partition, etc.) from the model and then reduce the state space for solution. But they usually require the model to satisfy a set of conditions hence have some limitations in their wide applicability. Furthermore, an important point to the modelling process is, whether or not the modeler can make a trade-off at the beginning of the modeling process, to those factors in consideration, for example, the “repre-

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tativeness” of the model vs. the complexity of the modelling process, the capability of the chosen method in specification vs. the easiness of the solution to the model, the accuracy of the result vs. the achieved efficiency in modelling and solution. Thus, the modeler can avoid or limit somewhat the later arriving challenges at an earlier stage of the modelling, before it is too late to take any costly postmortem actions to deal with the magnified problems. Having this in mind, the care should be taken to the beginning of the modelling process so as to limit the problem, e.g. the careful considerate selection of the model specification methods to avoid the state generation largeness, the proper use of model solution methods to get an approximate result but ensure certain confidence. The objective is for a modeler to seek to balance a reduction of the state space and the obtainment of a level of accuracy to the result.

## 5 Limiting Techniques in Modelling

To limit the state space explosion problem, we propose to use several techniques at the beginning of the modelling process, i.e. the model specification stage. As taking the dynamic issues into account in the network modelling and analysis, an very essential thing is to introduce the specification of those dynamic issues into the dependability model. This will definitely add some additional information to the system state. This crucial point was stated in [11] as “Control signals are considered as additional places of PN, which increases the number of places of a PN model and consequently the size of matrices”. Some later postmortem analyses to the state space generation also revealed that these “control” information contributed to the explosion of state space to an great extent. Accordingly, a first step to limit the state space explosion is to specify the system “state” (as intended) with minimal explicit information. The second technique is to limit the number of successive failures considered due to the fact that most of the well designed components have typically high availability and thus the probability of having (e.g. 3 or) more successive failures in the network is then relatively very low. The third technique is to just consider the most probable things, or in other words, disregard some less probable events when the transition rates among states differ largely in order of magnitude. These techniques, if used with proper tradeoffs in the modelling, can be applied at the beginning of the modelling process, i.e. the model design and specification stage, so that the modeler can avoid or limit state space explosion problem at an earlier stage.

## 6 An Application and Case Study

As an application of our extended dependability modelling methodology and proposed state space explosion limiting techniques, we consider to model the Internet dependability with OSPF routing protocol [12]. The Internet exists in order to transfer information from source nodes to destination nodes. Accordingly, one of the most significant functions performed by the Internet is the routing of traffic from ingress nodes to egress nodes. Internet intra-domain networks use OSPF as the routing protocol, which is a link-state protocol. Most of the existing methods in dependability modeling focus on the connectivity measures, and can not reflect the dynamic behaviors inside the network. We propose to use the behavioral model to incorporate the dynamic issues into the network dependability models. The dynamic issues incorporated in the network model include the link failures/repairs, routing patterns, time delays of route restoration and network convergence, and traffic handling issue due to link capacity constraints.

In order to demonstrate or verify in some sense the applicability of these methodologies and techniques to the real world, a case study is chosen for this purpose. An IP backbone network operated by UNINETT [13], who is the operator of Norway’s

national research network, is the network for the modelling. The three state space explosion limiting techniques are all used in the modelling process. The modelling results showed that significant reductions (i.e. a 98 ~ 99% reduction) to the original state space were achieved by making certain proper tradeoffs and employing some state space explosion limiting techniques, while certain level of accuracy and confidence to the solution results were also achieved from the modelling by using these techniques. Some initial results for this case study are also presented and discussed.

## 7 Concluding Remarks

In conclusion, we sought to employ some techniques to limit the state space explosion problem as our taking dynamic issues into account in network dependability modelling and analysis, and the achieved results were promising. Ongoing work include the refinements to the initial work, e.g. to realize the parameters from the network operator's statistics data and tune the models so as to get the realistic results. Future work include to apply the modelling methodology to more cases in order to verify its general applicability to a wider scope.

**Keywords:** Network Dependability, Modelling and Analysis, State Space Explosion, Dynamic Issues, Internet, OSPF Routing

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